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MAXIMUM ENERGY-RELEASE RATE OF MIXED-MODE CRACK PROBLEMS
AND ITS RELATION TO EXISTING FRACTURE CRITERIA •

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10 Chien H. Wu

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This final report on Grant No. DAAG-29-78-G-0030 summarizes the analytical investigations conducted at the University of Illinois at Chicago Circle during the period from June 15, 1976 to September 15, 1979. The two main topics of investigation were crack branching and effects of crack-parallel load. A list of papers published under ARO-D sponsorship is included.		

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1. Introduction

During the period from June 15, 1976 to September 15, 1979, analytical investigations on certain fracture related problems were conducted at the University of Illinois at Chicago Circle under Grant No. DAAG-29-78-G-0030. The two main topics of investigation were crack branching and effects of crack-parallel load. Substantial progress has been made in both areas.

Our results on the maximum-energy-release-rate problem have been well received by researchers in fracture mechanics, even though the same problem had been treated on many occasions prior to our investigation. The effects of a crack-parallel load have never been studied in the context of nonlinear elasticity, and our results are completely new.

Before proceeding to discuss the technical work completed under the grant, we would like to mention that three graduate students in the Department of Materials Engineering at the University of Illinois at Chicago Circle were supported in part during their graduate work by funds allotted to us under the grant. Their names are Donna J. Ciszewski, William J. Fenner and John Wilkinson. Mr. Fenner has just completed an M.S. thesis, and the work will be submitted for publication in the near future.

2. Research Findings

A. Crack Branching

The questions involved in this aspect of the investigation required the solution of the elasticity problem depicted in Fig. 1 where σ_{ij} are the uniform stresses applied at

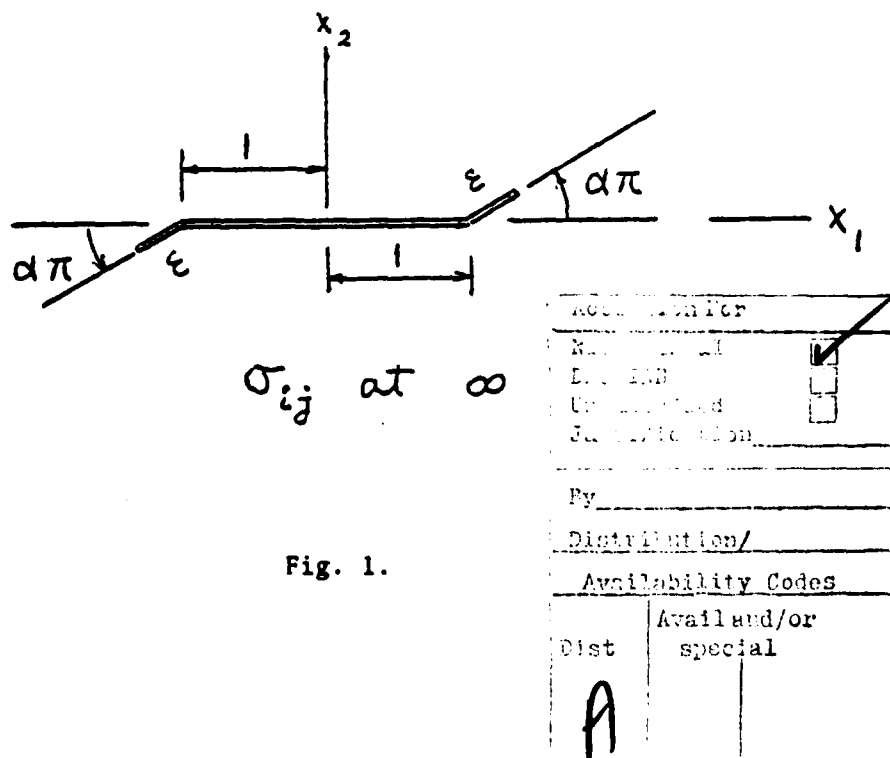


Fig. 1.

infinity, and the branch length ϵ is small when compared with the main crack length 2. In fact, the real objective was to determine the solution as well as several associated physical quantities as $\epsilon \rightarrow 0$.

Prior to our investigation, the problem had been studied in several publications [1-4].* These solutions are not very satisfactory in that they involve either certain analyticity assumptions or a numerical interpolation to attain the limit $\epsilon \rightarrow 0$. We have been able to avoid both in our solution procedure.

Our first attempt was to obtain a mapping function that maps the region exterior to the Z-shaped crack onto a half plane. This result was reported in the Appendix of (1).** It was obtained via the method of matched asymptotic expansions, and the explicit formula is accurate to the order of ϵ . To this end, it is perhaps worthwhile to mention that the same procedure may be applied to obtain mapping functions for other similarly singular configurations. Using this mapping function, the solutions for the plane and anti-plane problems were solved in (1). The solution for the anti-plane problem is exact and explicit while the solution for the plane problem requires the numerical integration of a single integral equation. These results were then generalized slightly to accommodate general loading conditions in (3).

The energy-release rate obtained in (1) was used to study the branching of a crack in a tension-compression specimen in (2), using the maximum-energy-release-rate criterion. As it was pointed out in our original proposal, neither the criterion nor the specific problem solved is new to the many researchers in the field of fracture mechanics. Our contribution then lies in the fact that we have obtained the solution to the mathematical question suggested by the maximum-energy-release-rate criterion with a procedure that has no apparent fault. In other words, our solution has provided us with the first opportunity to concentrate on examining the criterion itself rather than worrying about whether the answer to the question addressed by the criterion is correct.

In this connection, we have compared our results with those obtained from the maximum-stress criterion and minimum strain-energy-density criterion. The branching conditions implied by the maximum-stress criterion are very close to those implied by the maximum-energy-release-rate criterion. Furthermore, our extensive numerical results give the clear indication that a maximum energy release rate is always associated with a maximum K_I and a vanishing K_{II} .

*Numbers in brackets designate references listed in Section 4.

**Numbers in parentheses designate papers published under ARO-D sponsorship given in Section 3.

For the sake of documenting in a quantitative way the implications of the maximum-energy-release-rate criterion when applied to combined loading conditions, a rather exhaustive investigation was carried out in (4). The specific cases considered were:

- (a) Fracture locus for combined crack-perpendicular load and crack-parallel shear.
- (b) Fracture loci for plane biaxial loads.
- (c) Fracture locus for combined crack-parallel shear and anti-plane shear.
- (d) Fracture loci for combined anti-plane shear and plane uni-directional load.

Relevant tables and graphs may be found in (4).

The integral equation obtained in (1) was also solved asymptotically in (5), using the branch angle as a small parameter. The result was an explicit 3-term asymptotic solution. The various related asymptotic expressions were found to be quite accurate for branch angles as large as 72° .

In a very recent paper by Lo [5], a continuous distribution of dislocation was introduced to simulate the branches of a crack. An explicit Green's function was obtained and, as a result, Lo was able to extract analytically the ϵ -dependence from the ensuing numerical calculation. To substantiate the credibility of both his and our approaches, the maximum-energy-release-rate problem was re-examined by using both methods in (8). The two sets of numerical results were in perfect agreement. The effects of curving vis-a-vis branching were also studied in (8).

B. Effects of Crack-Parallel Loads

It is well known in the theory of linear elastic fracture mechanics that the stress-intensity factors at the tips of a crack are independent of a uniaxial load applied parallel to the crack. That this is so follows from the fact that the solution corresponding to such a load is just the trivial state of a uniaxial tension or compression. Physical intuitions, however, tend to suggest that a crack-parallel compression (tension) should have the effect of weakening (strengthening) the "stiffness" of a crack. Since the solution to a linear elasticity problem is unique, such a physical plausibility can only be examined by using a nonlinear analysis.

The first of such an analysis was reported in (6) where the material considered was of the harmonic type. The mathematics involved in analyzing a harmonic material is relatively simple in that the stress-

strain relation for an incremental state is isotropic. Two problems were considered in (8): (a) The buckling of a crack under a crack-parallel compression, and (b) A large crack-parallel load coupled with a small stress-intensity-producing load.

For the buckling problem, we found that there exists at least one and at most a finite number of buckling loads, depending on the type of harmonic material involved. Moreover, for each one of the possibly finite number of buckling loads, there exists a doubly infinite set of mathematically possible solutions, half of it symmetric with respect to the crack and the other half skew-symmetric. Of the symmetric half, only one that opens up the crack into an ellipse is physically possible. The rest of the symmetric half of the mathematical solutions indicate a penetration of the upper and lower surfaces of the crack into each other, and hence are discarded on physical grounds. All the skew-symmetric mathematical solutions are physically acceptable.

For the second problem, the stress-intensity factors were explicitly determined. They were found to depend on the crack-parallel load.

The same problems were studied again for general incompressible materials in (7). The results are qualitatively the same as those obtained for harmonic materials.

3. List of Papers Published under ARO-D Sponsorship

- (1) "Elasticity Problems of a Slender Z-Crack," J. of Elasticity, Vol. 8, No. 2, April 1978, 183-205.
- (2) "Maximum-Energy-Release-Rate Criterion Applied to a Tension-Compression Specimen with Crack," J. of Elasticity, Vol. 8, No. 3, July 1978, 235-257.
- (3) "The Plane Problem of a Semi-infinite L-Shaped Crack," Proceedings of the International Conference on Fracture Mechanics and Technology (edited by G. C. Sih and C. L. Chow), Noordhoff International Publishers, 1977.
- (4) "Fracture Under Combined Loads by Maximum-Energy-Release-Rate Criterion," J. Appl. Mech., Vol. 45, No. 3, Sept. 1978, 553-558.
- (5) "Explicit Asymptotic Solution for the Maximum-Energy-Release-Rate Problem," Int. J. Solids Structures, Vol. 15, No. 7, 561-566.
- (6) "Plane-Strain Buckling of a Crack in a Harmonic Solid Subjected to Crack-Parallel Compression," J. Appl. Mech. Vol. 46, No. 3, Sept. 1979, 597-604.

- (7) "Plane-Strain Buckling of Cracks in Incompressible Elastic Solids," J. of Elasticity, to appear.
- (8) "Curving Versus Branching," presented at the Third ASCE/EMD Specialty Conference; manuscript to be submitted to J. Appl. Mech.

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